NORTH SYDNEY GIRLS HIGH SCHOOL

HSC Mathematics Assessment Task 3
Term 2, 2009

Name:________________________________ Mathematics Class:_________

Time Allowed: 60 minutes + 2 minutes reading time

Available Marks: 60

Instructions:

- Questions are of equal value.
- Start each question on a new page.
- Put your name on the top of each page.
- Attempt all five questions.
- Show all necessary working.
- Marks may be deducted for incomplete or poorly arranged work.
- Each question will be collected separately.
- If you do not attempt a question, submit a blank page with your name and the question number clearly displayed.

Question	1ab	1cdef	2a	2bc	3abc	3d	4a	4b	5a	5b	Total
H3	/4										/4
H5		/8		/8	/8		/4			/8	/36
H8			/4	/		/4					/8
H9								/8	/4		/12
											/60

Question 1 (12 Marks) **Marks**

a) Evaluate the expression $e^5 + \log_e 50$ correct to 2 decimal places, **2**

b) Find the value of $\log_a (bc)^2$ given that $\log_a b = 2 \cdot 75$ and $\log_a c = 0 \cdot 25$. **2**

c) Each year a person's life seems only $\frac{9}{10}$ as long as the previous year from the second year of their life. What is the oldest a person could expect to feel, assuming they could live forever? **2**

d) Show that the curve $y = e^{x^2}$ is concave up for all values of *x*. **3**

e) Differentiate $y = \frac{e^x + 1}{2x}$ with respect to x. **2**

f) Find $\int (1+7x)^6 \, dx$. **1**

Question 2 (12 Marks) **Start a new page**

a) Consider the curve $y = \frac{1}{x^2}$ in the first quadrant.

i) Write an expression for *x* in terms of *y* for this function. **1**

ii) Find the area in the first quadrant between $y = \frac{1}{x^2}$, the *y* axis and the lines $y = 4$ and $y = 2$ (leaving your answer in surd form). **3**

b) Use the trapezoidal rule with 4 trapezia to calculate an approximation for **4**

$$\int_0^4 \frac{1}{x^2+1} \, dx.$$

c) Consider the function $y = e^{kx}$

i) Find the second derivative of this function. **1**

ii) Find all possible values of k for which $y = e^{kx}$ satisfies the equation **3**

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 35y = 0$$

Question 3 (12 Marks) **Start a new page**

a) Find $\int (3+e^x)^2\,dx$ **2**

b) Find $\int_1^4 \sqrt{x}\,dx$ **2**

c) A river 40 metres wide is measured for depth every 10 metres directly across its width.

These measurements, from bank to bank are given in the following table.

Distance from bank	0	10	20	30	40
Depth in metres	0	12·1	17·2	6·9	2

i) Find an approximation for the cross-sectional area of the river, using Simpson's rule and 5 function values **4**

d) A solid is generated when the region in the first quadrant enclosed between the curve $y=x^2$, the y axis and the line $y=4$ is rotated about the x axis.

i) draw a sketch of this information showing the region with shading. **1**

ii) Find the volume of the solid formed by rotating this region (leaving your answer in terms of π.) **3**

Question 4 (12 Marks) **Start a new page**

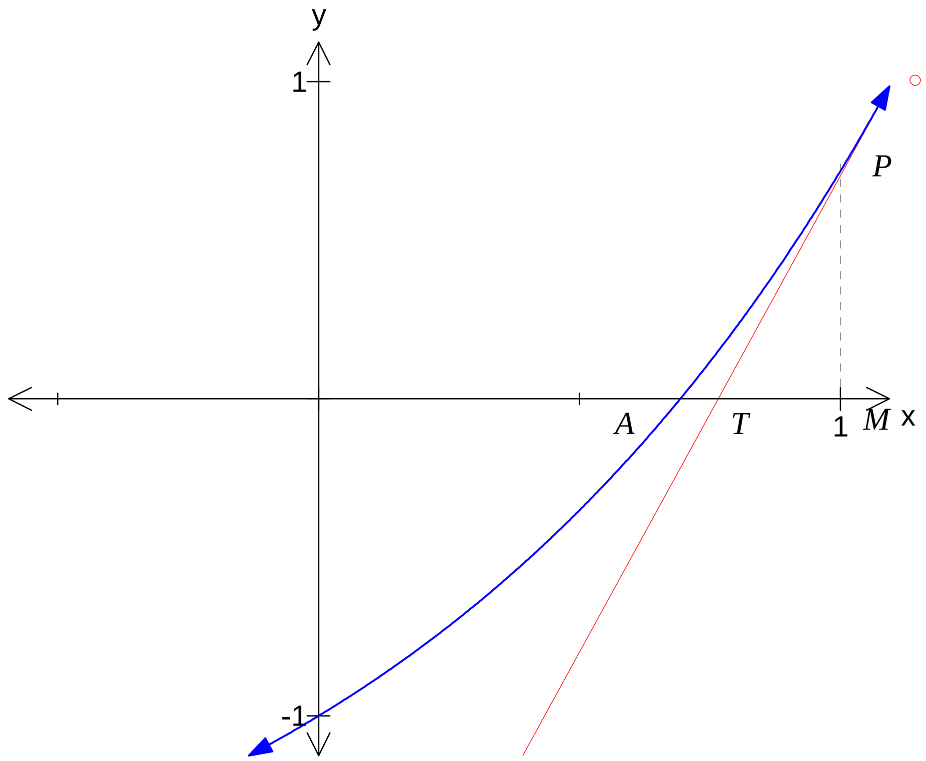
a) A geometric series has its nth term given by $T_n = (x-2)^n$.

i. Write out the first 3 terms of this series without expansion. **1**

ii. Find the range of values of x for which the series has a limiting sum. **1**

iii. Find this sum in terms of x in its simplest form. **2**

b) The diagram shows the graph of the function $y = e^x - 2$



The curve $y = e^x - 2$ cuts the x axis at A.

P is the point $(1, e-2)$ on the curve.

The tangent to the curve at P cuts the x axis at T

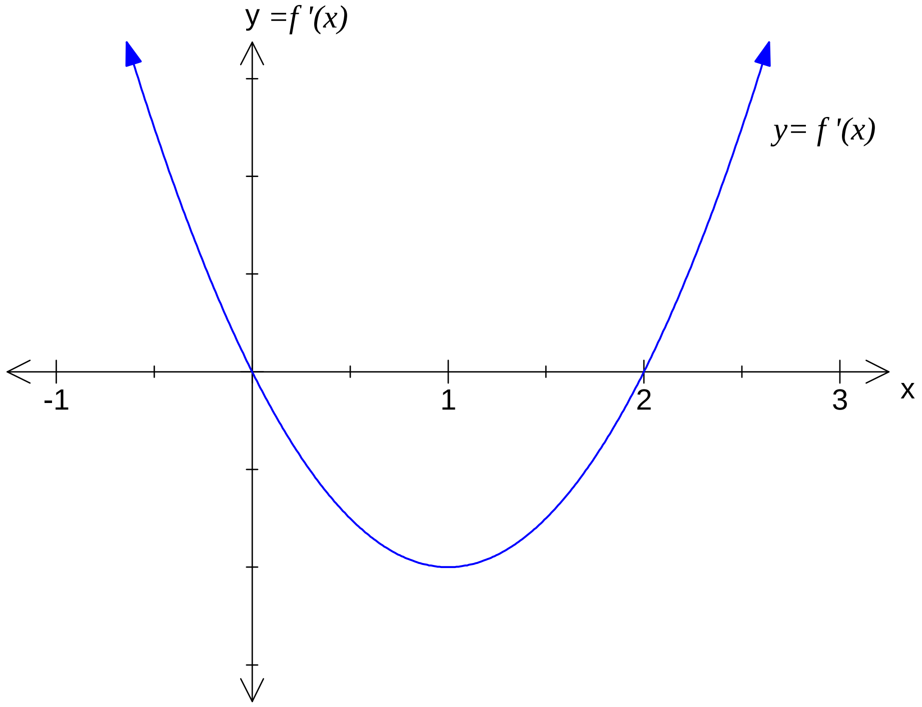
i. Find the x coordinate of the point A (in exact form). **1**

ii. Find the equation of the tangent at P and hence the x coordinate of the point T. **3**

iii. Find the area enclosed by the curve, the tangent at P and the x axis. **4**

Question 5 (12 Marks)   **Start a new page**

a) The diagram shows a sketch of the gradient function of the curve $y = f(x)$



It is known that this gradient function passes through the point $(1,-3)$

i) Show that $y = f'(x)$ has equation $f'(x) = 3x^2 - 6x$ **2**

ii) Find a possible equation for this curve and sketch its graph. **2**

b)Robyn invests \$1200 in an extra superannuation fund paying 4% pa compound interest.

At the beginning of the next year she deposits \$600 into this account which also earns 4% pa compound interest and continues to add \$600 to this account at the beginning of each year for a period of time (interest compounding annually at 4% pa).

i. What will be the value of Robyn's first 2 deposits when the first deposit has been invested for n years? **2**

ii. Derive an expression for the total accumulated amount of all her deposits at the end of n years (expressed in simplest form). **4**

iii. Robyn plans to withdraw all her funds when their total value reaches at least \$25000. How many deposits will she make in total including the first \$1200? **2**

66666666666666666666666666666666666666666666666666666666666666666666666666666666666666666

End of paper

TERM 2 2009 2UNIT (TASK 3)

Question 1

a) 150.0225

$\doteqdot 150.02$

b) $2\log_a(bc)$

$= 2(\log_a b + \log_a c)$

$= 2(2.75 + 0.25)$

$= 2 \times 3$

$= 6$

c) $S_\infty = \frac{a}{1-r}$

$= \frac{1}{1-9/10}$

$= \frac{1}{1/10}$

$= 10$

10 years old.

d) $y = e^{x^2}$

$\frac{dy}{dx} = 2x\,e^{x^2}$

$\frac{d^2y}{dx^2} = 2x \cdot 2xe^{x^2} + e^{x^2} \cdot 2$

$= 2(2x^2+1)e^{x^2}$

Concave up $\frac{d^2y}{dx^2} > 0$

$2e^{x^2} > 0$ for all x

$2x^2+1 > 0$ for all x

$\therefore$ Curve concave up for all x

e) $\frac{dy}{dx} = \frac{2xe^x - (e^x+1)2}{(2x)^2}$

$= \frac{2xe^x - 2e^x - 2}{4x^2}$

f) $\frac{(1+7x)^7}{7 \times 7} + c$

$= \frac{(1+7x)^7}{49} + c$

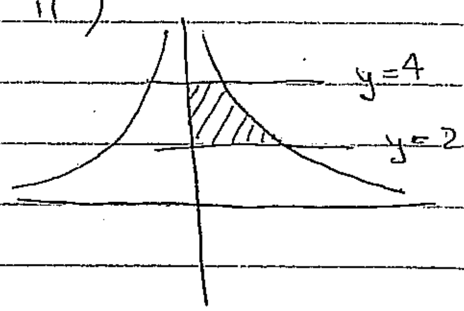
Question 2

a) i) $y = x^{-2}$

$y^{1/2} = x^{-1}$

$x = y^{-\frac{1}{2}}$

ii)



$A = \int_2^4 y^{-\frac{1}{2}}\,dy$

$= \left[2y^{\frac{1}{2}}\right]_2^4$

$= \left[2\sqrt{y}\right]_2^4$

$= 2\sqrt{4} - 2\sqrt{2}$

$= 4 - 2\sqrt{2}$

Area is $4-2\sqrt{2}$ units

Question 2.

b) $h = \frac{4-0}{4}$

$= 1$

x	0	1	2	3	4
y	1	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{17}$

$A \doteq \frac{h}{2}\left\{y_1 + y_5 + 2(y_2 + y_3 + y_4)\right\}$

$= \frac{1}{2}\left\{1 + \frac{1}{17} + 2\left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right)\right\}$

$= \frac{1}{2}\left\{1\frac{1}{17} + 2\left(\frac{4}{5}\right)\right\}$

$= 1\frac{28}{85}$.

c) $y = e^{kx}$

$\frac{dy}{dx} = ke^{kx}$.

$\frac{d^2y}{dx^2} = k^2e^{kx}$

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 35y = 0$

$k^2e^{kx} + 2ke^{kx} - 35e^{kx} = 0$.

$e^{kx}(k^2 + 2k - 35) = 0$

$(e^{kx} > 0)$ $\quad (k+7)(x-5) = 0$

$k = -7, 5$

Question three

a) $\int (3+e^{x})^2 \, dx$

$= \int 9 + 6e^{x} + (e^{x})^2 \, dx$

$= \int 9 + 6e^{x} + e^{2x} \, dx$

$= 9x + 6e^{x} + \frac{1}{2}e^{2x} + C$

b) $\int_1^4 \sqrt{x} \, dx = \int_1^4 x^{\frac{1}{2}} \, dx$

$= \left[\frac{2x^{3/2}}{3}\right]_1^4$

$= \frac{2}{3}\left[x\sqrt{x}\right]_1^4$

$= \frac{2}{3}(8-1)$

$= \frac{14}{3}$

c) $h = 10$

$A \doteq \frac{h}{3}\left\{(y_1 + y_5) + 2(y_3) + 4(y_2 + y_4)\right\}$

$= \frac{10}{3}\left\{(0+2) + 2(17\cdot2) + 4(12\cdot1 + 6\cdot9)\right\}$

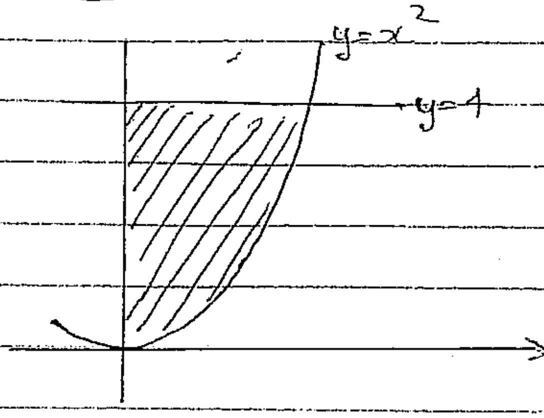
$= \frac{10}{3}\left\{2 + 34\cdot4 + 76\right\}$

$= 374\cdot\dot{6}$

Area is $374\cdot\dot{6}$ m^2

Question 3

d)



$$V = \pi \int_0^2 (4)^2\,dx - \int_0^2 (x^2)^2\,dx$$

$$= \pi \int_0^2 16 - x^4\,dx$$

$$= \pi \left[16x - \tfrac{1}{5}x^5\right]_0^2$$

$$= \pi \left(\left(32 - \tfrac{32}{5}\right) - (0 - 0)\right)$$

$$= \frac{128\pi}{5}$$

Volume is $\frac{128\pi}{5}$ units^3

Question Four

a) $T_n = (x-2)^n$

i) $(x-2)^1 + (x-2)^2 + (x-2)^3 + \ldots$

ii) $r = (x-2)$

$-1 < (x-2) < 1$

$1 < x < 3$

iii) $S_\infty = \dfrac{a}{1-r}$

$= \dfrac{(x-2)}{\cancel{1-(x-2)}}$

$= \dfrac{x-2}{3-x}$

Question 4

b) (i) $y = 0, \quad y = e^{x} - 2$

$0 = e^{x} - 2$

$e^{x} = 2$

$x = \log_e 2.$

(ii) $y = e^{x} - 2$

$\frac{dy}{dx} = e^{x}$

$x = 1, \quad \frac{dy}{dx} = e.$

$y - (e-2) = e(x-1)$

$y - e + 2 = ex - e$

$y = ex - 2$

$y = 0, \quad 0 = ex - 2.$

$ex = 2$

$x = \frac{2}{e}.$

(iii) $A = \int_{\log_e 2}^{1} e^{x} - 2 \, dx - \int_{\frac{2}{e}}^{1} ex - 2 \, dx.$

$= \left[e^{x} - 2x \right]_{\log_e 2}^{1} , \left[\frac{ex^2}{2} - 2x \right]_{\frac{2}{e}}^{1}$

$= \left((e-2) - \left(e^{\log_e 2} - 2\log_e 2 \right) \right) - \left(\left(\frac{e}{2} - 2 \right) - \left(\frac{e}{2}\left(\frac{2}{e}\right)^2 - \right. \right.$

$= \left(e - 2 - (2 - 2\log_e 2) \right) - \left(\frac{e}{2} - 2 - \left(\frac{2}{e} - \frac{4}{e} \right) \right)$

$= e - 4 + 2\log_e 2 - \frac{e}{2} + 2 - \frac{2}{e}$

$= \frac{e}{2} - 2 + 2\log_e 2 - \frac{2}{e}.$

Question five

a) i) $y' = ax(x-2)$

$(1,-3)$ $-3 = a(-1)$

$a = 3$

$y' = 3x(x-2)$

$y' = 3x^2 - 6x$

ii) $y = x^3 - 3x^2 + C$

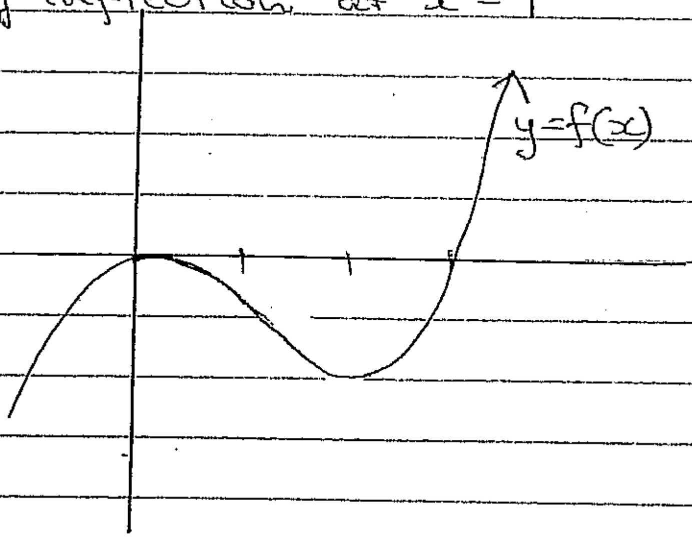
$= x^2(x-3)$

at $x=0$ Max t.p.

at $x=2$ Min t.p.

roots at $x=0$ and $x=3$.

point of inflection at $x=1$



5) (i) 1st deposit $\$1200\left(1+\frac{4}{100}\right)^{n}$

2nd deposit $\$600\left(1+\frac{4}{100}\right)^{n-1}$

first 2 deposits $\$\ 1200(1.04)^{n}+600(1.04)^{n-1}$

(ii) 3rd deposit amounts to $600(1.04)^{n-2}$

4th deposit amounts to $600(1.04)^{n-3}$

$$\text{Total amount} = 1200(1.04)^{n}+600(1.04)^{n-1}+600(1.04)^{n-2}+\cdots$$
$$+\cdots+600(1.04)^{2}+600(1.04)$$
$$=1200(1.04)^{n}+600(1.04)\left(1+1.04+1.04^{2}+\cdots+1.04^{n}\right)$$
$$=1200(1.04)^{n}+600(1.04)\left\{\frac{1\left((1.04)^{n-1}-1\right)}{1.04-1}\right\}$$
$$=1200(1.04)^{n}+600(1.04)\frac{\left(1.04^{n-1}-1\right)}{0.04}$$
$$=1200(1.04)^{n}+15000(1.04)\left(1.04^{n-1}-1\right)$$
$$=1200(1.04)^{n}+15000(1.04)^{n}-15000(1.04)$$
$$=16200(1.04)^{n}-15600$$

(iii)
$$25000=16200(1.04)^{n}-15600$$
$$16200(1.04)^{n}=40600$$
$$1.04^{n}=\frac{40600}{16200}$$
$$1.04^{n}=\frac{203}{81}$$
$$n=\log_{1.04}\left(\frac{203}{81}\right)$$
$$=23.425 \text{ years}$$

Robyn must make 24 deposits.